ALAP 2020
IX Congreso de la Asociación Latinoamericana de Poblacion
9 a 11 diciembre

EL ROL DE LOS ESTUDIOS DE POBLACIÓN TRAS LA PANDEMIA DE COVID-19 Y EL DESAFÍO DE LA IGUALDAD EN AMÉRICA LATINA Y EL CARIBE

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Sensitivity Analysis for the Brass P/F ratio method

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* The Brazilian Institute of Geographic and Statistics is exempt from any responsibility for the views, information, data and concepts expressed in the article, which are the sole responsibility of the authors

Introduction

CRVS systems are the natural data source for fertility estimation, but they suffer from coverage and quality problems, especially in developing countries. This might be due to lack of incentives to register a birth, which is aggravated when the child dies shortly after birth. Some of the births would be registered late, which can also affect fertility estimates (Moultrie 2013a).

These limitations have stimulated the development of demographic methods for fertility estimations, often based on censuses and surveys.

One of the earliest indirect methods for fertility estimation is the own-children method, which consists of a reverse-survival technique that uses the population of children in a census to estimate fertility in the recent past (Grabill and Cho 1965; Cho, Retherford, and Choe 1986). The main limitation of this method is related to the accuracy of the number of children enumerated in the censuses, which is likely to be undercounted.

Fertility can be also estimated directly through data containing birth histories collected in surveys such as the DHS. Another approach, perhaps the most used in developing countries, combines the recent and cumulated lifetime fertility measures routinely collected in censuses and surveys. These methods reconcile information from recent fertility, which is often underestimated, with the total parity by age group, which is thought to be more reliable. There are several procedures that use these idea, generally called P/F ratio methods (UN 1983; Moultrie 2013b). The most commonly used method of this kind is the Brass P/F ratio method (Brass 1964; UN 1983; Moultrie and Dorrington 2008), which will be discussed in detail below.

Demographers have estimated fertility rates by using indirect demographic techniques, primarily the Brass P/F ratio method, for decades. In Brazil, for instance, even though the limitations of this method have been widely known, the lack of an alternative data sources and some features of fertility change has led to a general consensus that these estimates reasonably describe the overall levels and trends in fertility in many contexts (Carvalho 1982; Berquó and Cavenaghi 2014). More recently, with the continuous decline of fertility levels and the rapid change in the age schedule, in addition to a greater availability of alternative data sources due to the improvement of vital registration systems, scholars have challenged the results of these techniques for the Brazilian context (Castanheira and Kohler 2015; Carvalho, Gonçalves, and Silva 2018).

This issue remains unsolved and there has been significant disagreement about the levels of fertility for the past decades, particularly for subnational levels. Fertility estimates using different methods and data sources have led to different results. Indirect demographic methods have several limitation, but CRVS systems in Brazil are also limited. Despite substantial improvements oven the last years, there are still a large proportion of births that are not registered, particularly in the less developed regions, which undermines their use without any adjustments. Furthermore, population estimates require long time series for periods when administrative records were wisely limited and sometimes inexistent.

PF Ratio Method

As discussed above, the most commonly used indirect technique to estimate fertility in contexts of defective vital registration systems is the P/F ratio method (Moultrie and Dorrington 2008). This method was first proposed by William Brass in the 1960s, initially for application to the African populations (Brass 1964; Brass and Coale 1968), and remains useful for estimating fertility in many countries.

The basic idea of the method is that, under certain assumptions, the number of children ever born, or parity (P_i), at an exact age i equals the sum of the period age specific fertility rates (F_i) to that age. Any difference in these two measures would be attributed to data quality problems, often an underreport of current fertility. The ratio for the age group $20-24$, for example, given by P_{20-24}/F_{20-24}, could then be used to adjust the fertility rates for all ages groups under the assumption that current fertility is underreported by a constant factor (Brass 1964; UN 1983).

The information used to calculate P_i and F_i comes from questions asked in censuses and surveys about children ever born and children born in the 12 months prior to the census.

Information on children ever born is often collected by questions such as: *How many children born alive have you had?*. This question requires no information on ages and dates so that there are no dating errors. On the other hand, there might be some imprecision in the number of children reported. It has been observed that children tend to be omitted, particularly by women aged 35 and over. Information for very young women are also thought to be more subject to reporting problems (UN 1983).

Current fertility is often collected in censuses by asking questions such as: *Have you given birth to any children in the past 12 months?*. This information is thought to be underreported, presumably because of a misperception of the reference period (Brass and Coale 1968; UN 1983). The way this question is asked has changed to allow for a more precise measure of the number of children born in the 12 months before the survey by asking about the date the last child was born: *What is the date of birth of the last children born alive you had?*. Brazilian censuses implemented this change in 1980. In fact, the estimated adjustment factor for the reported current fertility declined from 1.31 to 1.12 between 1970 and 1980, indicating an important improvement on this information. The adjustment factors remained similar for the next two censuses (Carvalho, Gonçalves, and Silva 2018).

Filtering the number of children born in the last 12 months improves the reporting of current fertility, as this avoids the error in the reference period, but this information remains underreported. One possible reason to this omission is related to the same phenomenon that leads to undercount of children in the censuses, that is, hard to count areas or population groups have a disproportional higher number of children. This may be related to the finding that newly formed households, which are more likely to have young children are more likely to be omitted in the census.

In summary, in the P/F ratio method, the age pattern of current ASFR obtained from surveys and censuses is accepted and the fertility level is adjusted by the average parity (number of children ever born) reported by young women. The idea of the method is to combine the measurements which are likely to be most reliable given the different nature of the deficiencies in the two sets of information (Brass 1964). Since the nature of errors tends to be different, this procedure may produce plausible estimates even when both sets of data are subject to errors (Brass and Coale 1968).

The method has thus two main assumptions in terms of the quality of the reported data: i) current fertility rates are underreported by a factor that is constant with age; ii) the reported number of children ever born is reliable.

Furthermore, there are three main conditions that need to be true for the method to be valid: i) constant fertility over time; ii) fertility of surviving woman is the same as fertility of those who died; iii) fertility of immigrant women is the same as non-immigrant fertility.

The P/F ratios are often calculated for all age groups between 15 and 49 because the pattern of the ratios by age may also reveal data quality problems and fertility trends. For adjustment purposes, the age group $20-24$ is by far the most used and recommended, due to data quality issues for women in the extremes of the age range, in addition to attempts to minimize the effects of fertility change (Brass 1964; UN 1983; Moultrie and Dorrington 2008). Thus, this paper concentrates on the analysis of the P/F ratio technique for women aged 20 to 24 at the date of the survey, which will be called P_2F_2[1].

To avoid confusion with the definition of age groups, let i be a constant representing the age to which the calculations are done and x be the actual age.

Formally, let

$$P_i(t) = \int_0^i f_x\,(t-i+x)\,dx$$

be the cumulative fertility up to age i at time t

$$F'_i(t) = \int_0^i f'_x(t)\,dx$$

be the cumulative observed period fertility up to age i at time t

$$F_i(t) = \int_0^i f_x\,(t)\,dx$$

be the cumulative true period fertility up to age i at time t

where $f_x(t)$ is the true ASFR at age x for year t. In this formulation of the method, the parity reported by woman at age i in census t, $P_i(t)$, is taken as the sum of the true historical fertility

[1] The index 2 has been used to refer to the second age group of women at reproductive ages, that is $20-24$ (Brass 1964; UN 1983), but all the results can be easily extended to other age groups.

rates for these woman, as the core assumption of the method is that this information is reliable. The current ASFR at age x, given by the reported fertility rates in the 12 months prior to the census t, $f'_x(t)$, relates to the true ASFR by a factor $\mathrm{PF}_i(t)$ as follows:

$$f_x(t) = f'_x(t) \times \mathrm{PF}_i(t)$$

$$F_i(t) = F'_i(t) \times \mathrm{PF}_i(t)$$

When $f'_x(t)$ is underestimated, which is often the case, $\mathrm{PF}_i(t)$ is greater than one.

For example, $P_{25}(2000) = \int_0^{25} f_x\,(1975 + x)\,dx$ is the parity, or retrospective fertility, of women aged 25 in 2000 and $F'_{25}(2000) = \int_0^{25} f'_x(2000)\,dx$ is the cumulative observed period fertility up to age 25 in 2000.

If the above-mentioned conditions are met, e.g., fertility below age i has been constant over time, $f_x(t - i + x) = f_x(t)$. Thus, $P_x(t) = F_x(t)$ and the P/F ratio $\frac{P_x(t)}{F_x(t)} = 1$. If the current fertility rates are omitted by a constant factor, the P/F ratio will be greater than 1 and will indicate the adjustment factor for the current fertility:

$$\frac{P_i(t)}{F'_i(t)} = \mathrm{PF}_i(t)$$

In practice, $P_i(t)$ is given by the average parity reported in the census at time t by women aged i:

$$P_i(t) = \frac{\mathrm{CEB}_i(t)}{K_i(t)}$$

where $\mathrm{CEB}_i(t)$ is the number of children ever born to women aged i at time t and $K_i(t)$ is the number of women from the age enumerated in the same census.

Similarly, $F'_i(t)$, in the discrete form is calculated as a sum of the ASFR, $f'_x(t)$, up to age i reported in the census at time t:

$$F'_x(t) = \sum_{x=0}^{i} f'_x(t)$$

$$f'_x(t) = \frac{\mathrm{B}_x(t)}{K_x(t)}$$

where $\mathrm{B}_c(t)$ is the number of births the 12 months before the census at time t to women aged x and $K_x(t)$ is the number of women as previously defined.

When the implementation of the method is done by five-year age groups, which is the most common approach, and the P_2F_2 factor is used, the information on current fertility at age $20 - 24$ needs an adjustment to be compatible with the average parity at the same age, as $\mathrm{CEB}_{20-24}(t)$ refers to the cumulated fertility experience of women in the age group $20 - 24$ at time t, including those in the beginning of the age group, e.g. at age 20. Thus, the process of cumulating current fertility considers the fertility of the entire group $15-19$ and only part of the fertility observed in the age group $20-24$:

$$f_{20-24}(t) = 5f_{15-19}(t) + k_{20-24}f_{20-24}(t)$$

where k_{20-24} is the multiplying factor for deriving the parity from ASFR, which also considers a half-year displacement backward in time as woman had their children, on average, about six months before the reported age in the census. There are many ways to estimate these factors. (Brass 1964)propose a model that relates the multiplying factor (k_{20-24}) to the ratio of the ASFR between the first two groups: $\frac{f_{15-19}(t)}{f_{20-24}(t)}$. The bigger the ratio, the younger the fertility age schedule and the bigger the adjustment factor.

Sensitivity Analysis

This section develops an analytical framework to evaluate how results of the application of the method are biased when one or more conditions of the method are not met, proposing adjustment factors to correct for these biases and to incorporate uncertainty in the estimates. Sensitivity analysis of indirect demographic methods, such as the P/F ratio, provides insights about the limitations of the methods and how sensitive they are to the violations of their assumptions and assist the estimates of the measures of uncertainty in demographic parameters in the next chapter.

Sensitivity analysis is this context is intended to evaluate how the results of a certain method would change when the assumptions are not satisfied. This procedure is useful to identify the most important assumptions required by the methods, indicating those that deserve special attention and those that are likely to have only minor impact in a certain context.

This analysis is also useful to propose adjustments in the methods if information on the factors affecting the results is available. In the specific context of this dissertation, this will be also useful to allow more precise prior distributions for the measures of uncertainty in demographic parameters.

Moultrie and Dorrington (2008) conducted a sensitivity analysis of the P/F ratio method, evaluating the impact of changes in fertility and mortality on the resulting adjustment factors and fertility estimates. The authors propose the use of simulations to overcome the analytical complexity of the basic equations of the model. The simulations were carried out to mimic a typical demographic transition change, particularly in the African experience.

The results show that the errors in the P/F ratio for the age group 20-24 are relatively small in the scenarios of changes in the fertility levels and age distributions. For most of the time, the errors are of the order of 5% or less, reaching a maximum of about 10%. The results under these hypothesis tends to overestimate fertility. The authors claim that in the context of the generally poor data in which these methods are normally applied, errors of this magnitude are not a major cause for concern.

Simulations were also carried out to test the sensitivity of the method to differential fertility between between survivors and non-survivors. This is operationalized by testing differential fertility between HIV-infected and HIV-uninfected women, which indicates that this has a trivial impact on the methods, even in an environment with a simulated highly generalized epidemic. That effect serves to attenuate the overestimation of the adjustment factors due to fertility changes, but errors are of the order of magnitude of only -0.5%.

The study of Moultrie and Dorrington (2008) offers important insights about the possible biases in the P/F ratio methods, and the order of magnitude of these errors. However, since they are based on simulations, the results are conditioned to the specific scenarios considered by the authors. Furthermore, they offer no possibilities for adjustments of the original proposition of the method.

In order to extend this analysis to other contexts and allow for adjustments in the original method, this section proposes an analytical sensitivity framework taking into consideration its main conditions.

Sections below develop a sensitivity framework to the hypothesis of differential fertility between survivors and non-survivors; differential fertility between migrants and non-migrants; and fertility change.

Mortality differential

The average parity calculated with the information of children ever born collected in censuses and surveys obviously refers to the fertility experience of the survivors of a particular cohort. If female mortality is low or there is no significant difference in fertility between survivors and non-survivors, then this information is a good proxy of cohort fertility up to that age.

It is generally assumed that the effect of mortality on the average number of children ever born is negligible, mainly because mortality is generally low for young women (UN 1983). In fact, this effect seems to be negligible even in extreme cases. As previously discussed, Moultrie and Dorrington (2008) report only trivial impact of differential fertility between HIV-infected and HIV-uninfected women on the method even in contexts of high HIV prevalence.

The formulation below offers some insights about the reasons mortality may have only a minor impact on biasing the results of the method. The results presented here are similar to those shown by Feehan and Borges (2018) for the sensitivity framework of the sibling survival method.

The average parity of women at the moment of the interview obviously reflects only those who survive, and is given by[2]:

$$P^s = \frac{\text{CEB}^s}{K^s}$$

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[2] the age group index is omitted in this section and all results refers to women aged $20-24$

The same quantity can be defined for women who died, although this is unobservable, reflecting the average parity they would have had if they had survived to the date of the interview:

$$P^d = \frac{\mathrm{CEB}^d}{K^d}$$

Now, let P^{ds} represent the aggregate average parity:

$$P^{ds} = \frac{\mathrm{CEB}^s + \mathrm{CEB}^d}{K^s + K^d}$$

Let

$$\pi^d = \frac{\mathrm{CEB}^d}{\mathrm{CEB}^d + \mathrm{CEB}^s}$$

be the proportion of births from women who died

$$\pi^s = \frac{\mathrm{CEB}^s}{\mathrm{CEB}^d + \mathrm{CEB}^s}$$

be the proportion of births from women who survived

Suppose P^d and P^s differ by a factor R^{ds}, for $R^{ds} > 0$:

$$R^{ds} = \frac{P^d}{P^s}$$

Based on these definitions, Feehan and Borges (2019) show that the ratio between the unobservable average parity of survivors and non-survivors (P^{ds}) and the average parity for the survivors P^s can be expressed as:

$$\frac{P^{ds}}{P^s} = \frac{R^{ds}}{\pi^d + R^{ds}(1-\pi^d)}.$$

Let PF^s be the adjustment factor of the age group $20-24$ calculated by the application of the P/F ratio method and PF^{ds} be the "true" adjustment factor that would have been observed if all the women had survived to the date of the interview. Since the denominator of the P/F ratio is the same, the only difference between PF^s and PF^{ds} is in the numerator (the average parity): P^s and P^{ds}.

Thus, the ratio between PF^{ds} and PF^s, which signals the bias in the P/F ratio method due to the unmet condition of equal fertility for women who died or survived, is given by:

$$\frac{\mathrm{PF}^{ds}}{\mathrm{PF}^s} = \frac{R^{ds}}{\pi^d + R^{ds}(1-\pi^d)}.$$

When $R^{ds} = 1$ (parity of survivors and non-survivors is equal) *or* when $\pi^d = 0$ (proportion of births from women who died is zero), $\mathrm{PF}^{ds} = \mathrm{PF}^s$, meaning that there is no bias in the result of the method.

This result shows that two conditions need to be simultaneously true to the unmet condition of independence between fertility and mortality have an impact on the results: i) fertility of women who do not survive to the interview differs significantly from those who survive; and ii) the proportion of women who die between the beginning of their reproductive lives and the age $20-24$ is considerable.

Since these are exceptionally rare conditions in real populations, this issue should not be a major cause for concern of researchers when using this method. In any case, researchers can easily use equation of sensitivity to mortality to asses the biases caused by fertility differentials in vital status with their own data.

In Brazil, even in the state with the highest mortality in 1980, the probability of a woman die between the age groups $15-19$ and $20-24$ was only about 1%. Even if a strong relationship between fertility and vital status is assumed (e.g. $R^{ds}=2$), the bias will be very small (0.5%):

$$\frac{\text{PF}^{ds}}{\text{PF}^{s}}=\frac{2}{0.01+2\times 0.99}=1.005.$$

Migration differential

The number of children ever born collected in the censuses, which is further compared to the current fertility in the P/F ratio method, refers to the fertility experience of all people enumerated in a certain geographic area, including the immigrants. Similarly to what occurs with fertility differentials by vital status, the P/F ratio may be biased if there is significant difference in fertility by migration status.

To illustrate this effect, imagine that if immigrants come from a region with high adolescent fertility, they will have higher parity than the cumulated period fertility in the region of destination, and the P/F ratio will then be overestimated.

The average parity of women at the moment of the interview reflects the fertility experience of both immigrant and non-immigrant women, and is given by:

$$P^{mn}=\frac{\text{CEB}^{mn}}{K^{mn}}$$

The same quantify can be defined for non-immigrant and immigrant women separately:

$$P^{n}=\frac{\text{CEB}^{n}}{K^{n}}\ P^{m}=\frac{\text{CEB}^{m}}{K^{m}}$$

where CEB represents the number of children ever born and K the population of women. The indices n, m and mn represent the non-immigrants, the immigrants and both groups combined, respectively.

Let

$$\pi^{n}=\frac{\text{CEB}^{n}}{\text{CEB}^{n}+\text{CEB}^{m}}$$

be the proportion of births from non-immigrant women.

$$\pi^m = \frac{\text{CEB}^m}{\text{CEB}^m + \text{CEB}^m}$$

be the proportion of births from immigrant women.

Suppose P^n and P^m differ by a factor R^{mn}:

$$R^{mn} = \frac{P^m}{P^n}$$

Following the same approach used in the previous section, let PF^{mn} be the adjustment factor of the age group $20 - 24$ calculated by the application of the Brass method and PF^n be the adjustment factor of the non-immigrants, which will be more consistent with the recent fertility experience of the enumerated population in a certain region. Again, since the denominator of the P/F ratio is the same, the only difference between PF^n and PF^{mn} is in the numerator (the average parity): P^n and P^{mn}.

Thus, the ratio between PF^n and PF^{mn}, which indicates the bias in the P/F ratio method due to the unmet condition of equal fertility according to the migration status, is given by:

$$\frac{PF^n}{PF^{mn}} = \left(\frac{\pi^m + R^{mn}(1-\pi^m)}{R^{mn}}\right).$$

When $R^{nm} = 1$ or when $\pi^m = 0$, $PF^n = PF^{mn}$, meaning that there is no bias in the result of the method.

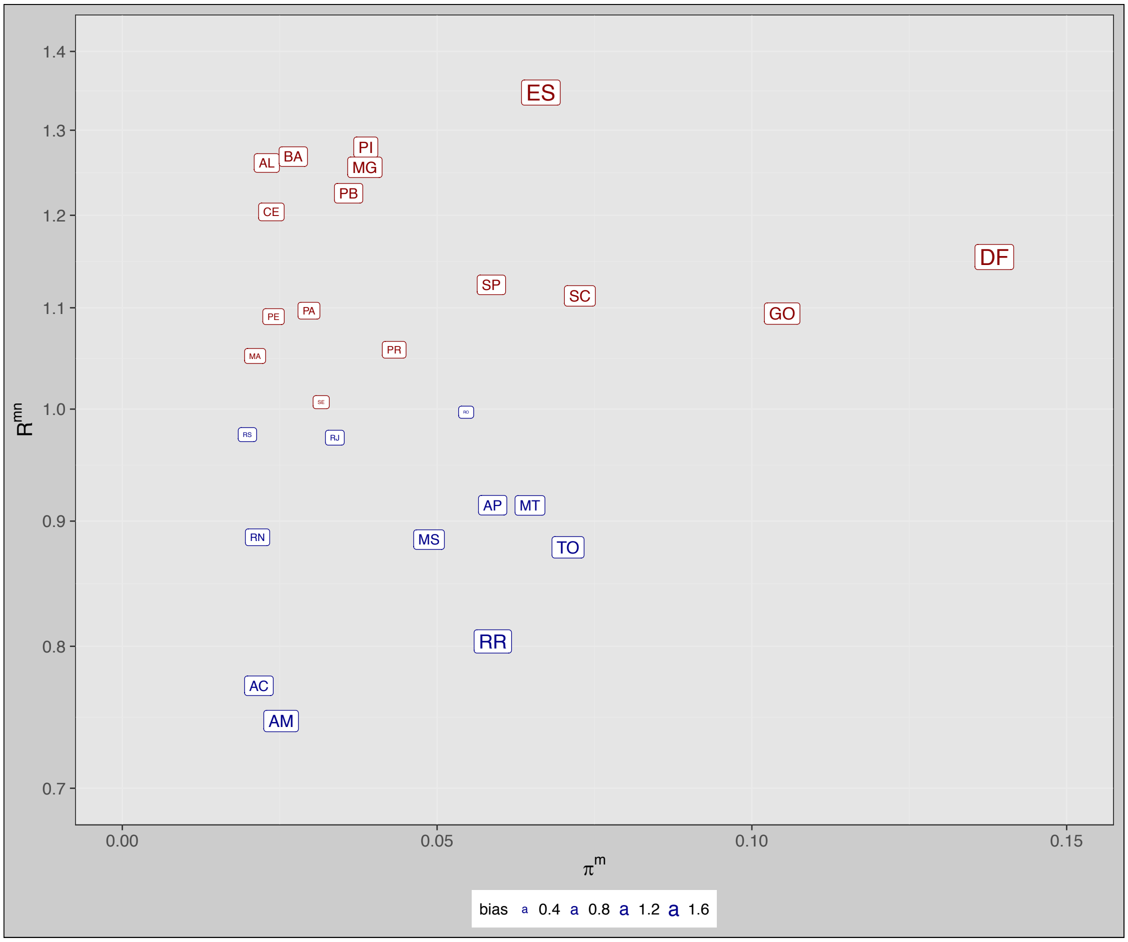
Similarly to what happens with mortality, this result shows that two conditions need to be simultaneously true to the unmet condition of independence between fertility and migration have an impact on the results: i) fertility of immigrant women differs significantly from the non-immigrants; and ii) the proportion of births from immigrant women is considerable.

Contrary to the mortality analysis, in which the proportion of woman who die at the beginning of the reproductive period is low, the proportion of immigrant at these ages, which in turn have different fertility rates, may be significant in many contexts, including subnational levels in Brazil.

It is not possible to derive from Brazilian censuses the number of births each woman had in the regions of origin or the region of destination. In order to approximate the quantities R^{mn} and π^m, the immigrants are defined as the woman aged $20 - 24$ who migrated less than 3 years before the census date. This groups would reflect the average fertility experience of woman who had their children outside the region of enumeration.

Some "immigrant" woman could have had their children recently in the region of destination, as well as the "non-immigrant" could have have their children more than 3 years ago in another region. However, the average parity and number of births provides a good approximating for illustrative purposes.

Figure 1 shows that the bias caused by the hypothesis of migration is higher than that caused by differential fertility by vital status, although it is still relatively low. The P/F ratio method overestimates fertility by slightly less than 2% in states such as DF and ES. In DF, this is mostly due to high proportion of immigrant women, whereas in ES the bias is mostly caused by the high fertility differential between immigrants and non-immigrants. The method underestimates fertility in a few states, such as RR.



Sensitivity of the P/F ratio method to migration, Brazilian states, 2010. Source: IBGE, 2010 Census.

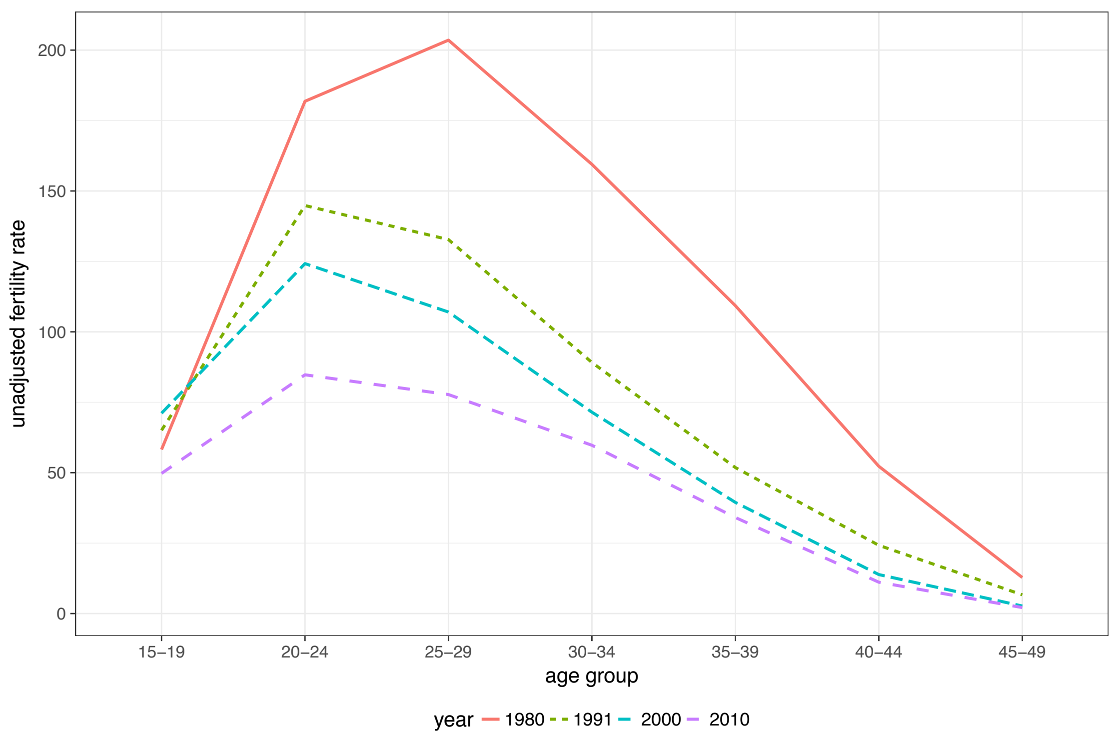
Fertility change

The last condition assessed in the sensitivity analysis is that of constant fertility. This is thought to be the strongest condition required by the method (UN 1983; Moultrie and Dorrington 2008).

In general, the condition of the method is that ASFR have remained constant over time. In fact, if the age group $20-24$ is used for calculating the adjustment factor, the requirement is that fertility for the two first age groups has remained constant in the recent past. It is common that fertility for the young women remains roughly constant in the beginning of the fertility transition period, despite rapid decline in old woman fertility.

In Brazil, as shown by Figure 2, adolescent fertility rates ($15-19$) increased in the periods 1980-1991 and 1991-2000, despite the rapid overall fertility decline. Fertility for the age group $20-24$ reduced in the same period, but at a slower pace than the observed for older ages. Thus, the almost constant fertility for the two younger age groups (increase in $ASFR_{15-19}$ offset by a moderate decline in $ASFR_{20-24}$) between 1980 and 2000, allowed the use of the P/F ratio method without marked biases (Carvalho, Gonçalves, and Silva 2018). On the other hand, $ASFR$ for these two age groups reduced more than 30% between 2000 and 2010, imposing a strong limitation to the use of the P/F ratio method for this period.

This issue has been a matter of debate in the past years. (Castanheira and Kohler 2015) identified the problem of the basic assumption violation for Brazil in 2010 and recommend that the method be possibly discontinued. (Cavenaghi and Alves 2016) also recognize the limitations, but argue that the method still presents reasonable results. (Carvalho, Gonçalves, and Silva 2018) discuss several alternatives. The final author's recommendation is the use of the adjustment factors for the year 2000 for 2010, with fertility estimated for the period of reference at about 2.5 years prior to the census.



ASFR for the years 1980, 1991, 2000 and 2010. Source: IBGE, Censuses of 1980, 1991, 2000 and 2010

The P/F ratio estimated by the Brass original method (PF^{B}) can be decomposed into two factors: i) the adjustment factor to correct the reported current fertility rates ($\widehat{\mathrm{PF}}$), which is what the method is trying to estimate; ii) a factor that estimates the effect of fertility change in the P/F ratio resulting from comparing cohort and period fertility (PF^{fc}). If fertility is constant over time, this second factor is 1.

$$\underbrace{\mathrm{PF}^{B}}_{\text{P/F ratio estimated by Brass original method}} = \underbrace{\widehat{\mathrm{PF}}}_{\text{underreport of current fertility effect}} \times \underbrace{\mathrm{PF}^{fc}}_{\text{fertility change effect}}$$

As previously discussed, the P/F ratio method assume constant fertility up to age i used for the calculation of the adjustment factor, so that $f_x(t)$ is constant over time and the P/F ratio is given by:

$$\mathrm{PF}_i(t) = \frac{P_i(t)}{F'_i(t)} = \frac{\int_0^i f_x\,(t)\,dx}{\int_0^i f'_x(t)\,dx}$$

The second factor of the equation calculates the effect of fertility change in the estimate of the P/F ratio method and is given by the ratio of the observed parity and the true period fertility rates:

$$\mathrm{PF}_i(t) = \frac{P_i(t)}{F_i(t)} = \frac{\int_0^i f_x\,(t-i+x)\,dx}{\int_0^i f_x\,(t)\,dx}$$

Following a similar approach used by (Schmertmann et al. 2013) to produce retrospective fertility estimates, let $f_x(t-i+x)$ be defined in terms of current fertility rates $f_x(t)$ and multipliers $\rho_x(t)$ that relate current (period) and retrospective (cohort) fertility rates, as follows:

$$\rho_x(t) = \frac{f_x(t-i+x)}{f_x(t)}$$

$$\mathrm{PF}_i(t) = \frac{P_i(t)}{F'_i(t)} = \frac{\int_0^i \rho_x\,(t) f_x(t)\,dx}{\int_0^i f_x\,(t)\,dx}$$

This ratio is the average ratio of past to present fertility rates, weighted by current fertility rates. This factor can be calculated if the past and present ratios are available. However, this is rarely available in contexts where this method needs to be applied. Thus, further approximations are necessary to transform equation above into parameters that can be estimated more easily.

The equation can be approximated further by the ratio between past to present fertility at the mean age of childbearing up to age i, μ_i at time $t-(i-\mu_i)$, where $(i-\mu_i)$ indicates the number of years prior to the census the experience of the cohort i refer to:

$$\mathrm{PF}_i(t) = \rho_{\mu_i}(t-i+\mu_i)$$

For example, assuming that $(i-\mu_i) = 2.5$ for the cohort aged $20-24$ in 2000, $\mathrm{PF}_{20-24}(2000) = \rho_{17-21}(2000-2.5)$, meaning that it refers to the ratio between fertility rates of the cohort $17-21$ in 1997.5.

A calibrated spline estimation procedure that interpolates detailed fertility schedules from age-group data (Schmertmann 2014) is used to calculate $(i-\mu_i)$, the average time since previous births. The ratio between fertility rates of the cohort, $\rho_{\mu_i}(t-i+\mu_i)$, is estimated based on the growth rate of the reported fertility rates between the census under analysis and the preceding census.

Sections below show the results of the application of this adjustment to Brazilian data from 1991 to 2010.

Omission of fertility among women under age 15

Finally, the application of the P/F ratio method often uses the traditional age groups of woman at reproductive ages, from 15 to 49. However, if fertility below age 15 is relatively high, this may bias the results.

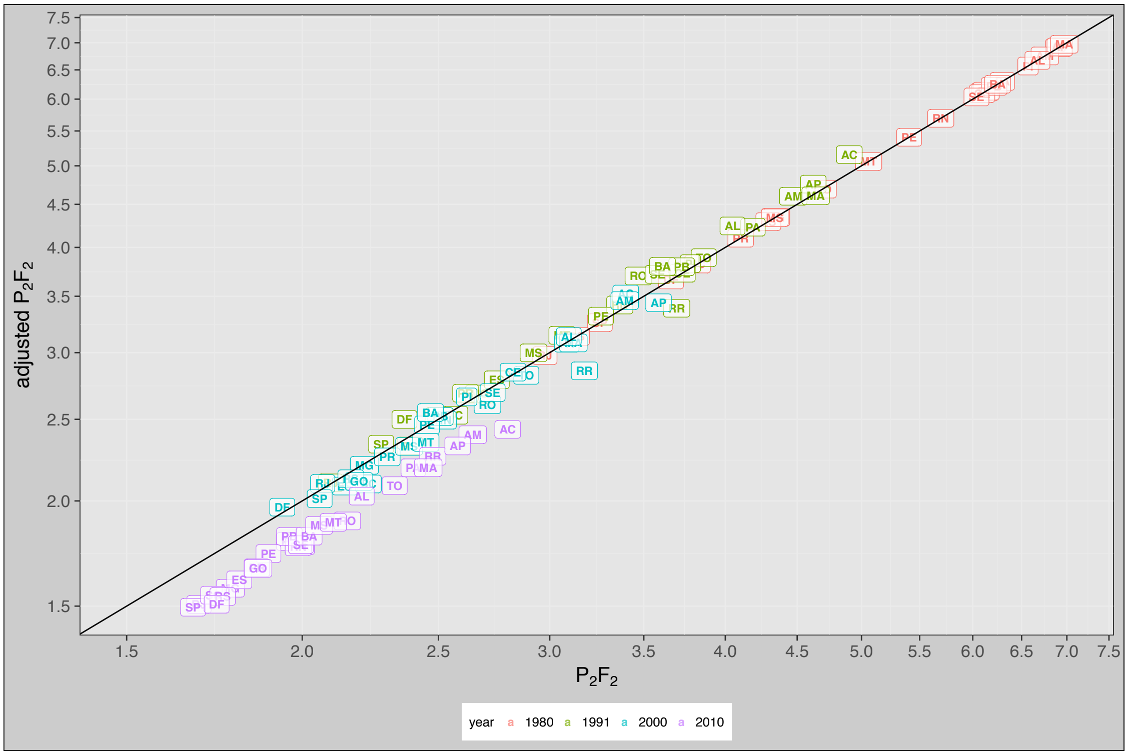
The fertility of women below age 15 are reported in the cumulated parity P_i, but this will not be taken into account if this group is not included in the calculations for the current fertility F_i. Thus, the adjustment factor will be overestimated.

For example, the fertility rates below age 15 reported in the 2010 Census in Brazil in a few states represents around 2% of the fertility rates up to age 24. This means that the P_{20-24}/F_{20-24} factor would be overestimated by 2%.

Fertility estimates for Brazil and states from 1980 to 2010 using the P/F ratio method

As sections above indicate, the only condition that can lead to important biases in the P/F ratio method in the Brazilian context is that of fertility change. Thus, the complete sensitivity framework derived in this section can be used in other contexts for to adjustment the final results of the methods. In this study, however, adjustments are made considering only this condition.

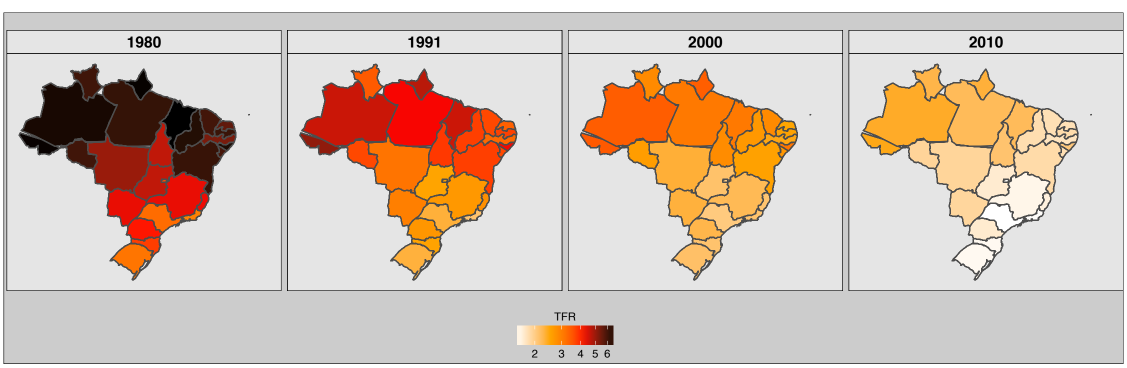
Figure 3 shows the comparison between adjusted an unadjusted TFR estimate by using the P/F ratio method. The figure shows that the only year when the adjusted TFR estimated by the method described above differ significantly from the original proposition is 2010. The adjusted rate consistently reduces fertility estimates for 2010. Is a few other cases, this occurs for other years as well, such as RR in 2000. For 1980, since there is no information on past fertility rates, both estimates are equal.



Comparison between adjusted an unadjusted TFR, Brazilian states, 1980, 1991, 2000 and 2010. Source: IBGE, Censuses of 1980, 1991, 2000 and 2010

Figure 4 shows the TFR by state, for the years 1980, 1991, 2000 and 2010 calculated by the P/F ratio method, using the adjustment ratio for the age group 20 – 24 (P2/F2).

The map indicates that fertility has declined steadily from 1980 to 2010. The TFR for Brazil, calculated by the P/F ratio method declined from 4.36 in 1980, 2.88 in 1991, 2.35 in 2000 and 1.71 in 2010. The regional differentials remain, despite the generalized fertility decline in all states.



Map of the TFR by state, for the years 1980, 1991, 2000 and 2010. Source: IBGE, Censuses of 1980, 1991, 2000 and 2010

Conclusion

Indirect demographic techniques remain relevant for many purposes. First, this is essentially the only way to estimate fertility for periods when CRVS were still incomplete. Second, despite improvements, administrative data still have problems in less developed regions, even for recent periods. Third, indirect demographic techniques, such as the P/F ratio method, have the advantage of using information from the same data source to estimate fertility. The use of CRVS also requires an estimated population of women at reproductive ages, which normally come from censuses. If the completeness of the census is different from the completeness of registered births, then fertility estimate will be biased. Finally, the fact that these questions are in censuses and surveys, which tend to have rich questionnaires, fertility for different population groups, e.g. education, income, migration and marital status, can be estimated.

This paper provides a novel sensitivity framework to the hypothesis of differential fertility between survivors and non-survivors; differential fertility between migrants and non-migrants; and fertility change. It also proposes an adjustment for estimating fertility in the context of adolescent fertility decline.

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